Does Mathematics Instruction for 3- to 5-Year Olds Really Make Sense?

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Running Head: Preschool Math?

DOES MATHEMATICS INSTRUCTION FOR 3- TO 5-YEAR OLDS REALLY MAKE SENSE?

Arthur J. Baroody

Over the past 25 years, researchers have accumulated a wealth of evidence that between 3 and 5 years of age, children actively construct a variety of fundamentally important informal mathematical concepts and strategies from their everyday experiences. Indeed, this evidence indicates that they are predisposed, perhaps innately, to attend to numerical situations and problems. It is important to note, too, that the mathematical ideas preschoolers construct are, in some cases, relatively abstract. Teachers of young children should be aware of their impressive informal mathematical strengths and recognize that it does make sense to involve them in a variety of mathematical experiences.

The purpose of this article is to review some of the recent research on young children's number and arithmetic concepts and skills. This research provides valuable insights into the extent of young children's mathematical learning; insights that can help us address the question of how best to provide mathematics experiences for young children.

What Mathematics Can 3- to 5-year-Olds Learn?

Over the course of the twentieth century, psychologists have come to dramatically different conclusions about preschoolers' mathematical competence. Indeed, their focus has shifted from trying to find what children *can't do* to trying to reveal what they *can do*.

Earlier Views

Behaviorism. Behavioral theorists have shaped the conventional wisdom about children's mathematical teaching and learning to this day (Ginsburg, Klein, & Starkey, 1998). Edward L. Thorndike (1922), for example, concluded that young children were so mathematically inept that "little is gained by [doing] arithmetic before grade 2, though there are many arithmetic facts that can [memorize by rote] in grade 1" (p. 198). According to association theorists, children

had to be rewarded (bribed) to learn mathematics, understanding was not central to learning useful mathematical skills, and students must be spoon-fed mathematics because they are uninformed and helpless. This view served as the rationale for the drill approach (Thorndike, 1922) and, years later, shaped the doctrine of direct instruction (Bereiter & Englemann, 1966). The lecture-and-drill method remains to this day the most widely used way to teach children mathematics in the primary grades.

Piaget's Constructivism. Jean Piaget's (1965) research offered a very different view of mathematical teaching and learning (see, e.g., Kamii, 1985). In his constructivist view, young children have a natural curiosity. For example, they have an inherent desire to find patterns and resolve problems, the essence of mathematics. For Piaget, the construction of mathematical understanding was the heart of real development. For example, reflecting on the part-whole relations underlying addition, such as a whole is the sum of its parts and greater than any single part, advances mathematical thinking, the memorization of number facts by rote does not. In Piaget's view, children actively construct their mathematical knowledge by interacting with their physical and social world. For instance, by listening to their parents, older siblings, and peers, children detect counting patterns, devise counting rules, and sometimes over-apply these rules, rather than passively absorb (imitate) the counting-word sequence they hear. A clear indication of this is rule-governed counting errors such as *fourteen, **fiveteen,** sixteen*, or *twenty-eight*, *twenty-nine*, ***twenty-ten*** (e.g., Baroody, 1998; Ginsburg, 1977). (It does not seem likely that many parents, siblings, or preschool teachers model and reward *fiveteen* or *twenty-ten*.) There is now abundant evidence to support the views of Piaget outlined above (see, e.g., Ginsburg and colleagues, 1998).

Interestingly, though, in Piaget's (1965) view, children so young are not capable of abstract concepts or logical thinking. Thus, he concluded, they are not capable of constructing a true concept of number or understanding of arithmetic.

Recent Developments

Over the last 25 years, psychologists have revealed a more optimistic portrait of preschool-

ers' mathematical competence. Summarized below are findings regarding concepts fundamental to number sense and an understanding of school (formal) mathematics. By understanding what young children know about these foundational concepts and can do with them, teachers can incorporate developmentally appropriate activities to nurture children's mathematical development.

Informal Concept of Numerical Equivalence. An ability to identify equivalent collections (e.g., recognizing $\bullet\ \bullet$, $\begin{smallmatrix}\bullet\\ \bullet\end{smallmatrix}$, $\begin{smallmatrix}\bullet & \\ & \bullet\end{smallmatrix}$, HH, ΘΘ, and νν all as pairs or two, recognizing $\bullet\ \bullet\ \bullet$, $\begin{smallmatrix}\bullet & \\ \bullet & \bullet\end{smallmatrix}$, $\begin{smallmatrix} & \bullet & \\ \bullet & & \bullet\end{smallmatrix}$, HHH, ΘΘΘ, and νννν all as trios or three, and so forth) is fundamental to understanding number. Research indicates that 3-year-olds can already recognize equivalence between small collections of objects or pictured objects—that is, visually match collections or pictures of 1 to 4 items (e.g., Huttenlocher, Jordan, & Levine, 1994; Siegal, 1973). For example, they can identify $\begin{smallmatrix}\bullet & \\ \bullet & \bullet\end{smallmatrix}$ and νννν as "the same" and νν as "not the same." Moreover, 4-year-olds, but not 3-year-olds, can make auditory-visual matches, such as equating the sound of three dings with the sight of three dots (Mix, Huttenlocher, & Levine, 1996) and accurately compare sequential sets with static ones, such as three jumps by a puppet or three light flashes with three dots (Mix, in press). Research further indicates that object-counting competence (e.g., the ability to count a collection or count out a collection of a particular size) appeared to be necessary for making auditory-visual or sequential-static matches, but not visual matches, of small collections (Mix, in press; Mix and colleagues, 1996).

What these results seem to indicate is that 3-year-olds have already developed a nonverbal representation of number. Whether this representation consists of a mental picture, mental markers (analogous to tallies), or something else is not entirely clear. Nor is it clear whether this representation is an exact one or an estimate. In any case, the key finding is that 3-year-olds already have a reasonably accurate way of representing and comparing small collections before they even learn to count.

Between $3\frac{1}{2}$ and 4 years of age, the development of their verbal and object-counting skills provides children a more powerful tool for representing and comparing numbers. In addition

to allowing preschoolers to make auditory-visual or sequential-static comparisons of small sets, their counting-based representation enables them to compare collections larger than four items. Specifically, by counting and visually comparing small collections, children can recognize the *same number-name principle*: Two collections are equal if they share the same number name, despite differences in the physical appearance of the collection (Baroody, 1998). Because it is a general (abstract) principle, young children can use it to compare any size collection that they can count.

Similarly by counting and visually comparing two unequal collections, preschoolers can further discover the *larger-number principle*: The later a number word appears in the counting sequence, the larger the collection it represents (e.g., *five* represents a larger collection than *four* because it follows *four* in the counting sequence). Once children can automatically cite the number after another in the counting sequence (e.g., The number after *four* is *five*), they can use the larger-number principle to *mentally* compare two *numbers* (e.g., Who is older, someone 9 or someone 8?—the 9-year-old because 9 comes after 8). This relatively abstract number skill has many everyday applications and can be used for even huge number (1,000,129 is greater than 1,000,128 because, according to our counting rules, the former comes after the latter). Typically, children can cite the number after another up to *ten* and can use this knowledge and the larger-number principle to mentally compare any two numbers up to *five* before they enter kindergarten (are $4\frac{1}{2}$ to $5\frac{1}{2}$ years of age). By the time they leave kindergarten (are $5\frac{1}{2}$ to 6 years old), children typically can compare any two numbers at least up to *ten*.

Informal Addition and Subtraction. An understanding of addition and subtraction is fundamental to success with school mathematics and everyday life. Recent research indicates that children start constructing an understanding of these arithmetic operations long before school. During the preschool years, they develop the ability to solve simple nonverbal addition or subtraction problems (e.g., Huttenlocher and colleagues, 1994). Such problems involve showing a child a small collection (1 to 4 items) covering it, adding or subtracting an item or items, and then asking the child to indicate the answer by counting out an appropriate number of disks.

For one item plus another item ("1 + 1"), for instance, a correct response would involve counting out a set of two disks rather than, say, one disk or three disks. In the Huttenlocher and colleagues' (1994) study, for example, most children who had recently turned 3-years-old could correctly solve problems involving "1 + 1" or "2 - 1" (that is, they could imagine adding one object to another or could mentally subtract one object from a collection of two objects). Most who were about to turn 4-years-old could solve "1 + 2," "2 + 1," "3 - 1," "3 - 2" as well, and at least a quarter could also solve "1 + 3," "2 + 2," "3 + 1," "4 + 1," "4 - 1," and "4 - 3." Thus, by the age of 4, children can mentally add or subtract any small number of items.

How do children so young manage these feats of simple addition and subtraction? They apparently can reason about their mental representations of numbers. For "2 + 1," for instance, they form a mental representation of the initial amount (before it is hidden from view), form a mental representation of the added amount (before it is hidden), and then can *imagine* the added amount added to the original amount to make the latter larger. In other words, they understand the most basic concept of addition—it is a transformation that makes a collection larger. Similarly, they understand the most basic concept of subtraction—it is a transformation that makes a collection smaller.

Later—but typically before they receive formal arithmetic instruction in school—children can solve simple addition and subtraction word problems (e.g., Huttenlocher et al., 1994), including those involving numbers larger than 4. How do they manage this? Basically, children decipher the meaning of the story by relating it their informal understanding of addition as a "make-larger" transformation or their informal understanding of subtraction as a "make-smaller" transformation (e.g., Baroody, 1998; Carpenter, Hiebert, & Moser, 1983). They then—at least initially—use objects (e.g., blocks, fingers, or tallies) to model the meaning (type of transformation) indicated by the word problem. Consider the following problem: *Rafella helped her mom decorate five cookies before lunch. After lunch, she helped decorate three more cookies. How many cookies did Rafella help decorate altogether?* Young children might model this problem by counting out five blocks to represent the initial amount, counting out three more blocks to

represent the added amount, and then counting all the blocks put out to determine the solution.

Research further reveals that children invent increasingly sophisticated counting strategies to determining sums and differences (e.g., Baroody, 1998; Carpenter and colleagues, 1983; Resnick & Ford, 1981). At some point, children abandon using objects and rely on verbal counting procedures. To solve the problem above, for instance, they might count up to the number representing the initial amount ("one, two, three, four, five") and continue the count three more times to represent the amount added ("six is one more, seven is two more, and eight is three more—eight cookies altogether"). One shortcut many children spontaneously invent is to start with number representing the initial amount instead of counting from *one*: "Five; six is one more, seven is two more, and eight is three more—eight cookies altogether" (Baroody, 1995).

As with making number comparisons, children's informal addition is initially relatively concrete (in the sense that they are working nonverbally with real collections or mental representation of them) and limited to small collections of four or less. Later, as they master and can apply their counting skills, they extend their ability to engage in informal arithmetic both in terms of more abstract contexts (word problems and, even later, symbolic expressions such as 2 + 1 = ?) and more abstract numbers (namely, numbers greater than four).

Part-Whole Relations. The construction of a part-whole concept (an understanding of how a whole is related to is parts) is enormously important achievement (e.g., Resnick & Ford, 1981). For example, it is considered to be a conceptual basis for understanding and solving missing-addend word problems such as Problems A and B below and missing-addend equations such as ? + 3 = 5 and ? - 2 = 7.

ν **Problem A.** *Angie bought some candies. Her mother bought her three more candies. Now Angie has five candies. How many candies did Angie buy?*

ν **Problem B.** *Blanca had some pennies. She lost two pennies playing. Now she has seven pennies. How many pennies did Blanca have before she started to play?*

Young children's inability to solve missing-addend word problems and equations has been

taken as evidence that they lack a part-whole concept (e.g., Riley, Greeno, & Heller, 1983). Some have interpreted such evidence as support for Piaget's (1965) conjecture that the pace of cognitive development limits what mathematical concepts children can and cannot learn and have concluded that instruction on missing addends is too difficult to be introduced in the early primary grades (Kamii, 1985).

The results of several recent studies suggest otherwise (e.g., Sophian & Vong, 1995). Sophian and McCorgray (1994), for instance, gave 4-, 5-, and 6-year-olds problems like Problems A and B above. Problems were read to a participant and acted out using a stuffed bear and pictures of items. When reference was made to the initial unknown amount, the participant was shown a round box covered by an envelope. When reference was made to adding objects, a picture of their objects were shown to the child and then put in the envelope (out of sight). For problems involving subtraction, a picture of the objects taken was removed from the box, shown to the child, and then placed out of sight. When the result was mentioned, the participant was shown a picture of the corresponding items. Although 5- and 6-year-olds typically had great difficulty determining the exact answers of such problems, they at least gave answers that were in the right direction. For Problem A, for instance, children knew that the answer (a part) had to be less than five (the whole). For Problem B, for example, they recognized that the answer (the whole) had to be larger than seven (the larger of the two parts). These results suggest that 5- and 6-year-olds can reason (qualitatively) about missing-addend situations and, thus, have a basic understanding of part-whole relations.

Equal Partitioning. Subdividing a collection or other quantities into equal-sized parts (equal partitioning) is the conceptual basis for division, measurement, and fractions. Research has shown that many children of kindergarten age can, for instance, can respond appropriately to fair-sharing situations or problems such as: *Three sisters Martha, Marta, and Marsha were given a plate of six cookies by their mom. If the three sisters shared the six cookies fairly, how many cookies would each sister get?* (e.g., Davis & Pikethly, 1990; Hiebert & Tonnesen, 1978). Some children solve this type of problem by using a divvying-up strategy: Count out (six) objects to represent the

amount, then deal out the cookies one at a time to each pile (to each of three piles), repeat the process until all the objects have been passed out (repeat the process again), and then count the number of items in each pile to determine the solution (count the two blocks in one of the piles). In effect, research suggests that even the operation of division can be introduced to children as early as kindergarten.

Informal Fraction Addition and Subtraction. Perhaps most surprising of all is the research indicating that preschoolers can understand simple fraction addition and subtraction. Mix, Levine, and Hutlenlocher (1999) presented 3-, 4-, and 5-year-olds nonverbal problems that involved, for instance, first showing half of a circular sponge and then putting it behind a screen, next showing half of another circular sponge and then putting it behind the screen also, and finally presenting four choices (e.g., one-quarter of sponge, one-half of a sponge, three-quarters of a sponge, and a whole sponge) and asking which was hidden by the screen. The 3-year-olds were correct only 25% of the time (i.e., responded at a chance level—no better than could be expected by random guessing). The 4- and 5-years olds, though, responded at an above chance level. For instance, over half were correct on problems involving "$\frac{1}{4} + \frac{1}{2}$," "$\frac{1}{4} + \frac{3}{4}$," "$\frac{1}{2} - \frac{1}{4}$," and "$1 - \frac{1}{4}$."

Conclusion

Recent research indicates that preschoolers do have impressive informal mathematical strengths in a variety of areas. Given this and their natural inclination for numerical reasoning, it does make sense to involve them in mathematical instruction. A key question is: How can teachers best provide them with engaging, appropriate, and challenging mathematical activities?

How Should Preschoolers be Taught Mathematics?

In this section, I first discuss a new approach for teaching mathematics and then delineate some key implication for early childhood mathematics instruction.

The Investigative Approach

Consistent with constructivist theory and its supporting evidence, the National Council of Teachers of Mathematics or NCTM (1989, 1991) has recommended shifting from a traditional instructional approach to an approach that better fosters the mathematical power of children. As will be evident below, this new approach is consistent with the teaching guidelines outlined in the revised edition of *Developmentally Appropriate Practice in Early Childhood Programs* (Bredekamp & Copple, 1997).

What is Mathematical Power? Mathematical power has three components. The first is a positive disposition to learn and use mathematics. This includes the beliefs and confidence needed to tackle challenging problems. As these are learned, teachers need to help children develop the belief that everyone—to a significant degree—is capable of understanding mathematics and solving mathematical problems. The second element of mathematical power is understanding mathematics. This includes appreciating how school mathematics relates to everyday life, seeing the connections among mathematical concepts, and linking procedures to their conceptual rationale. In order to promote meaningful learning, then, teachers must help children (a) relate school-taught symbols and procedures to their informal, everyday experience, (b) consider how different ideas like addition and subtraction are related (e.g., adding one can be undone by subtracting one and 3 - 2 = ? can be thought of as 2 + ? = 3), and (c) learn the whys as well as the hows of mathematics. The third part of mathematical power is developing an ability to engage in the processes of mathematical inquiry. This includes making and testing conjectures, finding patterns in the world around us (inductive reasoning), problem solving, and logical reasoning (deductive reasoning). An especially important (but often overlooked) process is communicating about mathematics. To promote the ability to engage in mathematical inquiry, teachers need to find challenging (but developmentally appropriate) problems and encourage students to discuss their suggestions for solving it and their solutions with their others, including their peers.

Why is Mathematical Power Important? A positive disposition toward mathematics underlies, for example, the confidence and perseverance necessary to tackle challenging problems

and life-long learning of mathematics. Understanding greatly facilitates remembering (retention) and applying (transfer) of mathematics (e.g., Brownell, 1935; Hatano, 1988; Hiebert & Carpenter, 1992; Rittle-Johnson & Alibali, 1999). (Meaningful learning requires less drill and practice than does learning by rote. Moreover, because children can apply what they understand, students can make connections and learn new material more easily on their own.) Problem solving and other inquiry skills are increasingly necessary in an increasingly complex world.

How Can Instruction Foster Mathematical Power? A traditional instructional approach unfortunately, robs children of mathematical power (Baroody, 1998). To better foster mathematical power, the NCTM (1989, 1991) has recommended that instruction be purposeful; meaningful, and inquiry-based—what has been called the "investigative approach" (Baroody, 1998). In this approach, instruction begins with a worthwhile task, a task that is interesting, often complex, and *creates a real need* to learn or practice mathematics. Experiencing mathematics in context is not only more interesting to children but more meaningful—both of which make learning it more likely (Donaldson, 1978; Hughes, 1986). In the investigative approach, a teacher helps children build on what they already to know to learn new concepts or procedures. By connecting new information or a problem to existing knowledge, children are far more likely to understand it. The instruction involves children in making conjectures, solving problems, inductive and deductive reasoning, and communicating their ideas, findings or conclusions. There is no better way to become proficient at these inquiry skills than to engage in real mathematical inquiries. Planning activities that are purposeful, meaningful, and inquiry-based are at the heart of good early childhood teaching practices.

Implications for Early Childhood Mathematics Instruction

The investigative approach seems particularly well suited for preschool children and their mathematics instruction. Below are some suggestions for making mathematics experiences for young children purposeful, meaningful, and inquiry based.

Purposeful Instruction. Teachers need to find or devise worthwhile tasks that create a real

need for learning and practicing mathematics. There are a variety of ways to make mathematics instruction purposeful, such as using everyday situations, children's questions, games, and children's literature.

- *Everyday preschool activities provide numerous opportunities to learn or practice mathematics* (see, in particular, Baroody, 1998; Fromboluti & Rinck, 1999; Kamii, 1985). For instance, in preparing for snack time, table setters for each table can be asked to count the number of children in their group present that day to determine the number of place settings needed (counting a collection) and then put out a carton of milk, piece of fruit, paper plate, utensil, or whatever is needed for each child in the group (counting out a collection or one-to-one matching). Note that such tasks might involve other skills such as addition (e.g., My group usually has five, but Clayton is sick today, so we have...). Note also that teachers should not assign such tasks to children who are not developmentally ready (e.g., the atypical 4-year-old who cannot verbally count to at least *five* or *six*) and should leave enough time to help those who are developmentally ready but who have not learned or mastered a needed skill.

- *Children's questions can provide invaluable teachable moments*. For example, Diane asks, "My birthday is next week, how old will I be?" Will I be older than Barbara? A teacher could respond by commenting, "Class, Diane has some interesting questions she needs help with. If she is 3-years-old now, how can she figure out how old she'll be on her next birthday?" Note that the teacher asked the group how to solve the problem ("She could count and see what number comes after *three*"), not simply to give the solution ("She will be four"). The teacher could then follow-up by posing a problem involving both number-after and number-comparison skills: "If Barbara is 5-years-old and Diane is 4-years-old, how could we figure out who is older?" Answering their own real questions can provide a powerful incentive to engage in mathematical inquiry and to explore or practice mathematical content. Furthermore, such conversations about mathematics can provide teachers with a rich source of information about children's present and emerging understandings of number and arithmetic.

- *Games provide a natural, entertaining and structured way to explore or practice mathematics.*

Play is a natural way of exploring their world and mastering skills for coping with it (Bruner, 1986). Playing math games can be an enjoyable way of raising interesting question or practicing key mathematical skills. Furthermore, teachers can choose or design math games to raise a particular issue or practice a particular skill (see, in particular, Baroody, 1989, 1998; Kamii, 1985). For example, while playing *Race Car*; Ari rolled a large balsa wood die, and the side with five dots came up. He immediately recognized  as five (pattern recognition) and moved his car five spaces on the race track (counting out a collection of spaces). Bret then rolled the die and five dots came up again. He counted the dots (verbal counting and counting a collection) and, beginning with the space on which is car is resting, counted five spaces. "Hey," Bret complained, "I started on the same space as Ari, rolled a five like him, and now I'm behind him. How can that be?" Through a discussion guided by the teachers, the players concluded that Bret counted as "one" the space he was on. The teacher asked Bret, "If you rolled a 'one,' what would that mean?" Bret answered, "I could move one space." "Show me," asked the teacher. Bret started to count as "one" the space he was on again but now realized that would mean his car would not advance. To help Bret (and others) remember how to count out spaces correctly, the teacher recommended starting their counts with zero (using the whole number sequence "0, 1, 2, 3, . . . ," rather than the natural number sequence "1, 2, 3, . . . "). In brief, the game provided verbal- and object-counting practice, created a real need for discussing and correcting a common counting error, and brought about an opportunity for introducing the whole number series.

- *Children's literature can provide another rich and entertaining source of problems and content learning* (see, e.g., Burns, 1992; Fromboluti & Rinck, 1998; Theissen & Mathias, 1992; Whitin & Wilde, 1992). Consider, for example, *The Doorbell Rang* by Pat Hutchins (© 1986 by Greenwillow Books). The story begins with a mother presenting a brother and sister a plate of cookies and instructing them to share them (fairly). Before reading on, a teacher could ask how this could be done and how many cookies each child would get. Pairs of children could be given, say, poker chips to model the situation, and the group could then share their strategies. With any

luck, at least one pair will suggest using an equal partitioning (divvy-up) strategy. As a follow-up activity, the children could role play this and other stories that involve mathematical concepts.

Meaningful Instruction. Teachers should focus on promoting meaningful learning, rather learning by rote. To foster meaningful learning, teachers should promote and build on children's informal mathematical knowledge and focus on helping them see patterns and relations.

- *Foster and build on children's informal mathematical knowledge.* In particular, it is important to encourage preschoolers' use of verbal, object, and finger counting to represent, reason about, and operate on numbers. Recall that counting is a powerful tool for extending young children's nonverbal numerical and arithmetical competencies. Opportunities to learn and practice counting skill should be abundant, and the use of counting solutions should be praised.
- *Focus on helping children see patterns and relations.* Instruction on mastering the verbal counting sequence, for example, should concentrate on helping preschoolers discover counting patterns (e.g., the "teens" are largely a repetition of the original sequence + "teen": *six* + *teen*, *seven* + *teen*, . . .) and the exceptions to these patterns (e.g., "Although '*fiveteen*' is a good name for the number after *fourteen*, most people call it '*fiveteen*'"). Playing an error-detection game, where a muppet or "addled" adult character tries to count and children have to help out by pointing out errors, such as ". . . *nineteen*, ***tenteen***," can be an enjoyable way for them to learn and practice counting rules and their exceptions.

Inquiry-Based Instruction. By involving children in inquiry-based instruction, teachers can foster a positive disposition. This includes promoting beliefs such as I can solve mathematical problems and mathematics, at heart, is an attempt to find patterns in order to solve problems. Inquiry-based instruction can also promote meaningful learning when children, for instance, discover a mathematical relation or listen about their peers' discoveries. Finally, it can help inquiry skills such as the ability to reason about and solve real or challenging problems. Involving children in inquiry-based instruction means that teachers should encourage them to

discover and do as much for themselves as possible (Polya, 1981). This does not mean that teachers should simply allow children to engage in free play all the time. Learning is more likely to occur if wiser adults or older children mediate younger children's experiences (Anderson, 1997; Blevins-Knabe & Musun-Miller, 1996; Bruner, 1986; Durkin and colleagues, 1986; Lave, Murtaugh, & de la Rocha, 1984; Lawler, 1990; Leino, 1990; Rogoff, 1990; Saxe, Guberman, & Gearhart, 1987; Vygotsky, 1968). Some ways teachers can mediate learning are noted below.

- *Regularly pose worthwhile tasks (e.g., thought-provoking and interesting questions or problems) and encourage children to answer or solve them themselves.* For example, a teacher asked Suzie, a kindergartner, what she thought the largest number was. She quickly answered, "A million." The teacher then asked what the number after a million might be. The girl thought for a moment and responded, "A million and one." Asked what she supposed the next number might be, Suzie answered quickly, "A million and two. So there is no biggest number." The teachers questions, in effect prompted this child to reflect on her knowledge of numbers, apply her knowledge of counting rules to continue the counting sequence past a million, and then deduce from this experience that the counting sequence in theory could go on forever (construct a concept of infinity).

- *In general, prompt reflection rather than provide feedback.* When children have difficulty arriving at a solution or arrive at an incorrect, provide hints, ask questions, or otherwise promote their thinking, rather than simply giving them the correct solution. For example, Kamie concluded that five and two more must be six. Instead of telling the girl she was wrong and the correct sum was seven, her teacher asked, "How much do you think five and one more is?" After Kamie concluded it was six, she set about recalculating five and two more. Apparently, she realized that both five and one more and five and two more could not have the same answer. In brief, the teachers question prompted her to reconsider her first answer.

- *Encourage peer-peer dialogue.* Other children can sometimes explain informal mathematical ideas or strategies to a child better than an adult can. Furthermore, sharing ideas with other

children can help a child clarify their own thinking. Indeed, because it can often result in disagreements and disagreements can prompt reconsidering their ideas, peer-peer dialogues can be an invaluable way of advancing mathematical thinking.

In conclusion, preschoolers are capable of mathematical thinking and knowledge that may be surprising to many adults. Teachers can support and build on this informal mathematical competence by engaging them in purposeful, meaningful and inquiry-based instruction. Although using the investigative approach will require imagination, alertness, and patience by teachers, its reward can be increasing significantly the mathematical power of children.

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Suggestions for Further Reading

For a more complete discussion of why the investigative approach makes sense and how to implement it, see the following:

- *Fostering Children's Mathematical Power: An Investigative Approach to K-8 Mathematics Instruction* (Baroody, with Coslick, 1998). This resource can be ordered from Lawrence Erlbaum Associates by telephone (800-926-6579), e-mail (orders@erlbaum.com), or FAX (201-236-0072).
- *Mathematical Power: Lessons from a Classroom* (Parker, 1993). This book can be ordered from Heinemann by telephone (800-541-2086) or FAX (800-847-0938).

For ideas about teaching specific content in a purposeful, meaningful, and/or inquiry-based manner, see the aforementioned references and, for example, the following resources:

- *Children's Mathematical Thinking: A Developmental Framework for Preschool, Primary, and Special Education Teachers* (Baroody, 1987). This reference can be ordered from Teachers College Press by telephone (800-576-6566) or FAX (802-864-7626).
- *Early Childhood Mathematics* (Smith, 1997). This book can be ordered from Allyn and Bacon by calling 800-666-9433.
- *Early Childhood, Where Learning Begins, Mathematics: Mathematical Activities for Parents and Their 2- to 5-Year-Old Children* (Fromboluti & Rinck, 1999). This reference can be ordered from the U.S. Department of Education by writing ED Pubs, P.O. Box 1398, Jessup, MD 20794-1398 or by calling (toll free) 1-877-4ED-Pubs.
- *A Guide to Teaching Mathematics in the Primary Grades* (Baroody, 1989). This resource, like most of the volumes listed here, can be ordered on-line through Amazon.com.
- *Mathematics in the Early Years, Birth to Five* (Copley, 1999). This reference can be ordered from the National Council of Teachers of Mathematics by phone (800-235-7566), FAX (703-476-2970), or e-mail (orders@nctm.org).
- *Mathematics Their Way* (Baratta-Lorton, 1976). This guide can be ordered from Addison-Wesley Publishing Co. by telephone (800-552-2759) or FAX (800-333-3328).

- *Young Children Reinvent Arithmetic: Implications of Piaget's Theory* (Kamii, 1985). This book can be ordered from Teachers College Press.